Map Building without Localization by Dimensionality Reduction Techniques

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Abstract

This paper proposes a new map building framework for mobile robot named Localization-Free Mapping by Dimensionality Reduction (LFMDR). In this framework, the robot map building is interpreted as a problem of reconstructing the 2-D coordinates of objects so that they maximally preserve the local proximity of the objects in the space of robot's observation history. Not only traditional linear PCA but also recent manifold learning techniques can be used for solving this problem. In contrast to the SLAM framework, LFMDR framework does not require localization procedures nor explicit measurement and motion models. In the latter part of this paper, we will demonstrate "visibility-only" and "bearing-only" localization-free mappings which are derived by applying LFMDR framework to the visibility and bearing measurements respectively.

1. Introduction

Most of the recent studies on mobile robot map building have employed the problem formulation of *simultaneous localization and mapping* (SLAM) which states the problem of estimating both robot states and object positions from a series of sensor measurements(Thrun, 2002). In fact, efficient algorithms for solving SLAM have been developed and used in many mobile robot products. We have no objection to the significance and value of these SLAM studies. However, we should not jump to the conclusion that other robotic map building paradigms than SLAM are meaningless. Especially, we would like to raise the following two fundamental questions about the robotic mapping research:

- First, isn't it possible to estimate the positions of objects (or features) directly from observation data without explicitly estimating the robot's pose, whereas mapping and localization are treated as inseparable in SLAM ?
- Secondly, isn't it possible to build maps only with weaker prior knowledge than the usual measurement and motion models, whereas those models are indispensable in SLAM ?

Based on these motivations, this paper proposes an alternative framework for robotic mapping named Localization-Free Mapping by Dimensionality Reduction (LFMDR). Intuitively, LFMDR is based on an idea that closely located objects tend to share similar histories of being sensed by a robot. More specifically, LFMDR attempts to find 2-D coordinates of objects so that they maximally preserve the local proximity of them in the space of robot's observation history. To obtain a low-dimensional representation (2-D coordinates) of objects from the high-dimensional observation data, not only traditional linear methods such as PCA but also non-linear dimensionality reduction techniques so called manifold learning methods can be utilized.

Compared with SLAM, LFMDR as a robotic mapping framework has remarkable features such as independence from self-localization of a robot, and lower requirements for prior knowledge. That is to say, it is able to build a map without estimating the robot's pose nor using measurement and motion models explicitly. Nevertheless, for the present, we must admit the cost of these advantages is not small. LFMDR is basically an off-line process, whereas most of SLAM algorithms work online. In addition, it requires a larger amount of observation data than SLAM at the expense of abandoning motion and measurement models.

In the latter half of this paper, we present *visibility-only* and *bearing-only* localization-free mappings which

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are derived by applying LFMDR framework to visibility information and bearing measurements of objects, respectively.

2. Related Works

2.1. SLAM

To clarify the characteristics of LFMDR which is explained in the next section, we briefly reconsider the formulation of SLAM(Thrun et al., 2005). Though SLAM is applicable to various map representations such as occupancy-grid and topological (graph-based) maps, we consider only feature-based maps here. This means a *map* is represented by 2-D coordinates of a finite number of objects or features. We hereafter use the following notation.

- $\boldsymbol{x}_t$: robot's pose (state) at time t.
- $m = \{\boldsymbol{\xi}_1, \cdots, \boldsymbol{\xi}_M\}$: map (positions of M objects)
- $\boldsymbol{u}_t$: robot motion command executed at time t
- $\boldsymbol{y}_t$: sensor measurements obtained at time t

The relationships between the variables are expressed by the state transition and observation equations:

$$\boldsymbol{x}_t = f(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t-1}, m) + \boldsymbol{v}_t \quad (1)$$
$$\boldsymbol{y}_t = g(\boldsymbol{x}_t, m) + \boldsymbol{w}_t \quad (2)$$

where, $\boldsymbol{v}_t$ and $\boldsymbol{w}_t$ stand for disturbance and observation noise, respectively. Note that these state transition and observation equations can be represented in the form of probabilistic motion model, $p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t-1}, m)$ and measurement model $p(\boldsymbol{y}_t|\boldsymbol{x}_t, m)$. Then, the goal of SLAM is to estimate both the history of robot's state $\{\boldsymbol{x}_{1:N}\}$ up to time N and map or 2-D coordinates of M objects $m = \{\boldsymbol{\xi}_1, \cdots, \boldsymbol{\xi}_M\}$, when the histories of observations $\{\boldsymbol{y}_{1:N}\}$ and commands $\{\boldsymbol{u}_{1:N-1}\}$ are given. Specifically, this is conducted by estimating the posterior distribution $p(\boldsymbol{x}_{1:N}, m|\boldsymbol{y}_{1:N}, \boldsymbol{u}_{1:N-1})$ or finding $\boldsymbol{x}_{1:t}$ and m which maximize it (MAP estimation). As is widely known, several approaches have been developed for solving SLAM problem including EKF applied to the extended state $[\boldsymbol{x}_t, m_t]$ (Leonard & Feder, 1999), alternate estimation by EM algorithm (Thrun et al., 1998), and Rao-Blackwellized particle filter (Montemerlo et al., 2002). In any case, these SLAM methods assume that motion and measurement models, i.e., functions f and g in equations 1,2 are explicitly provided beforehand, and have to estimate the robot's state $\boldsymbol{x}_t$ at each time step even if we only want to obtain a map m. In contrast, LFMDR does not require these assumptions.

2.2. Dimensionality Reduction

Dimensionality Reduction (DR) is a main topic in multivariate analysis whose purpose is to find a low-dimensional representation of high-dimensional input data. DR has also been a central theme in a variety of fields such as pattern recognition, datamining, and so on. Principal Component Analysis (PCA) is the most basic and widely used DR technique and its relationships with Multidimensional Scaling (MDS) and Singular Value Decomposition (SVD) is known. In addition to the linear DR methods, non-linear DR techniques called spectral manifold learning algorithms(Saul et al., 2006) have been actively studied in recent years.

These dimensionality reduction techniques have been successfully applied to various learning problems including classification, regression, clustering, and system identification. Especially, several studies attempting to apply DR techniques to the localization and mapping problems have been reported recently(Ham et al., 2005; Bowling et al., 2005; Ferris et al., 2007). They proposed methods of estimating low-dimensional representations of sequential states (or traces) of robots or mobile devices from temporal sequences of high-dimensional measurement vectors. These works demonstrated the possibility of *localization without mapping*, and can be regarded as a counterpart of our argument in this paper that *mapping without localization* is possible. In other words, their methods treated the measurements at different times as data points to which DR techniques are applied, whereas we treated the historical observations about different objects as data points. Another example of applying DR to map building can be found in (Brunskill & Roy, 2005), which applied mixture-PPCA to range measurements to extract low-dimensional geometric features (line segments). (Pierce & Kuipers, 1997) also used PCA to obtain low-level mappings between robot's actions and perceptions. While these two works used the DR techniques for estimating some "local" structures which are essential for mapping, we used them for recovering a global map of objects.

3. Mapping by Dimensionality Reduction

In this section, we formalize the idea of LFMDR. To do this, we make a few assumptions about the map building problem we consider here. First of all, we consider the feature-based mapping, and ignore the issue of data association by assuming all objects in the environment are uniquely identifiable by the robot's sensors. We also assume that absolute positions of

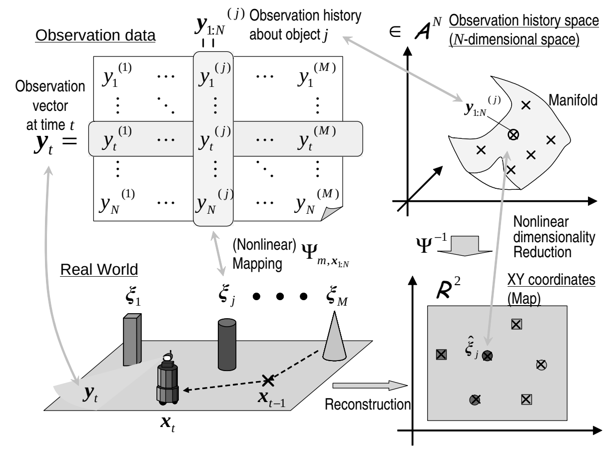

Figure 1. Mapping by Dimensionality Reduction of Observation History Data

some (three at least) objects are known in advance. We call these objects *anchor objects*. Next we assume that an observation vector $\boldsymbol{y}_t$ is composed of elements each of which corresponds to an object as follows,

$$\boldsymbol{y}_t = [\boldsymbol{y}_t^{(1)}, \boldsymbol{y}_t^{(2)}, \cdots, \boldsymbol{y}_t^{(M)}]^T \quad (3)$$

The last and most important assumption is that the measurement model $g(\boldsymbol{x}_t, m)$ can be approximately decomposed into submodels relating to individual objects as follows,

$$g(\boldsymbol{x}_t, m) \approx [g_{m,\boldsymbol{x}_t}(\boldsymbol{\xi}_1), \cdots, g_{m,\boldsymbol{x}_t}(\boldsymbol{\xi}_M)] \quad (4)$$

This implies that an observation relating to the j-th object $\boldsymbol{y}_t^{(j)}$ is approximately dependent only on the object's location $\boldsymbol{\xi}_j$ given the map m and current robot's position $\boldsymbol{x}_t$. In other words, it is assumed that $\boldsymbol{y}_t^{(j)}$ is a measurement of some spatial relationship (such as distance, direction, etc.) between the robot and j-th object. Thanks to the last two assumptions, a feature vector for each object $\boldsymbol{y}_{1:N}^{(j)}$ can be computed from the robot's historical observation data.

Under these assumptions, we can imagine there is a (non-linear) mapping between $\boldsymbol{y}_{1:N}^{(j)} \in \mathcal{A}^N$ and $\boldsymbol{\xi}_j \in \mathcal{R}^2$, where $\mathcal{A}$ stands for the space of $\boldsymbol{y}_t^{(j)}$.

$$\begin{aligned} \boldsymbol{y}_{1:N}^{(j)} &= \begin{bmatrix} \boldsymbol{y}_1^{(j)} \\ \vdots \\ \boldsymbol{y}_N^{(j)} \end{bmatrix} = \begin{bmatrix} g_{m,\boldsymbol{x}_1}(\boldsymbol{\xi}_j) \\ \vdots \\ g_{m,\boldsymbol{x}_N}(\boldsymbol{\xi}_j) \end{bmatrix} \\ &\equiv \Psi_{m,\boldsymbol{x}_{1:N}}(\boldsymbol{\xi}_j) \end{aligned} \quad (5)$$

Now consider that neither function g nor mapping $\Psi_{m,\boldsymbol{x}_{1:N}}$ is provided explicitly, but still we want to obtain a map m or a low-dimensional representation of objects $\boldsymbol{\xi}_1, \cdots, \boldsymbol{\xi}_M$ from the historical observation data $\boldsymbol{y}_{1:N}^{(1)}, \cdots, \boldsymbol{y}_{1:N}^{(M)}$. From this viewpoint, map building can be regarded as a process of extracting a low-dimensional representation for M objects from M N-dimensional historical observation vectors. This is exactly an issue of dimensionality reduction described earlier. Figure 1 illustrates the concept of mapping by dimensionality reduction.

In summary, the framework of map building procedure based on this idea is described as follows,

1. The robot explores the environment and collects a set of observation history data $\boldsymbol{Y}_{1:N}$ up to time N.
2. The observation data $\boldsymbol{Y}_{1:N}$ is decomposed into M column vectors $\{\boldsymbol{y}_{1:N}^{(1)}, \cdots, \boldsymbol{y}_{1:N}^{(M)}\}$ which correspond to the observation histories about M objects.
3. Apply your favorite DR technique to the set of vectors $\{\boldsymbol{y}_{1:N}^{(1)}, \cdots, \boldsymbol{y}_{1:N}^{(M)}\}$ or normalized one $\{\tilde{\boldsymbol{y}}_{1:N}^{(j)}\}_{j=1,\cdots,M}$ if necessary, and obtain a set of 2-dimensional vectors $\{\boldsymbol{z}_j\}_{j=1,\cdots,M}$.
4. Find an Affine transformation that minimizes the mean distance error of anchor objects, then apply it to $\{\boldsymbol{z}_j\}_{j=1,\cdots,M}$ and obtain a set of estimated object positions $\{\hat{\boldsymbol{\xi}}_j\}_{j=1,\cdots,M}$.

This formulation of map building based on dimensionality reduction has several remarkable features that SLAM does not have. First, the history of robot's pose $\boldsymbol{x}_{1:N}$ does not appear in the estimation process explicitly. In other words, a map (a set of object locations) is directly estimated from the observation history without self-localization. This is why we call it "localization-free" mapping. In a sense, the robot's pose $\boldsymbol{x}_t$ at each time is treated implicitly as a "dimension" of the robot's observation history space. Another point is that LFMDR requires much less prior knowledge on measurement and motion models. More specifically, LFMDR only requests that the observation $\boldsymbol{y}_t$ and measurement model $g(m, \boldsymbol{x}_t)$ meet the assumptions above. It is unnecessary to know what g is exactly.

On the other hand, a major drawback of LFMDR compared with SLAM is that it is basically an off-line algorithm, which means the map is reconstructed every time a new observation is obtained. This property will be disadvantageous if there is a need to consider moving objects. Another potential drawback is that it generally requires a larger amount of observation data than SLAM at the expense of abandoning high quality information sources such as motion and measurement models. Moreover, LFMDR in its original form does assume that the observation data $\boldsymbol{Y}_{1:N}$ has no missing elements. If it needs to deal with incomplete observation data (as is often the case in the real world), it has to perform the dimensionality reduction along with estimating the missing values by some method such as EM algorithm.

4. Visibility-only and Bearing-only Mappings

In this section, we present two instances of LFMDR - *Visibility-Only* Localization-Free Mapping (VOLFM) and *Bearing-Only* Localization-Free Mapping (BOLFM). They are derived by applying the LFMDR framework to visibility information and bearing measurements of objects, respectively. Both of them are tested in a simulated environment.

4.1. Common Settings

The simulated environment is a square region whose side length is 2.5[m] containing $M = 50$ randomly placed objects. Each object is cylinder-shaped with diameter of 48[mm]. Four of them are anchor objects whose absolute positions are given in advance.

At each observation position, the robot chooses its next moving direction randomly within the range of $\pm\frac{\pi}{4}$ and proceeds 200[mm] in that direction. It avoids collisions with objects and walls by changing its direction reactively when it approaches them. Every 200 (in BOLFM) or 500 (in VOLFM) observations, LFMDR is applied to the historical observation data, and maps are built. We conducted 25 runs by randomly changing the initial poses 5 times for 5 different layout patterns. Each setting (decided by DR method and its parameter value) was evaluated by averaging the results of all runs. To evaluate the accuracy of built maps, we employed both *quantitative* and *qualitative* criteria – (a) *Mean Position Error* (MPE) and (b) *Mean Orientation Error* (MOE) which is defined by the percentage of triangles (formed by arbitrary three objects) whose orientations (i.e., clockwise or anti-clockwise) are inconsistent with the ground truth maps.

4.2. Compared DR Techniques

This time we used the following 8 DR techniques.

1. **Linear PCA** (LPCA)
2. **SMACOF** (de Leeuw, 1977) : SMACOF is a metric MDS technique that locally minimizes a loss function called *raw stress* defined as:

$$\mathcal{E}_{SMACOF} = \sum_{i<j} w_{i,j}(\delta_{i,j} - \|\boldsymbol{z}_i - \boldsymbol{z}_j\|)^2 \qquad (6)$$

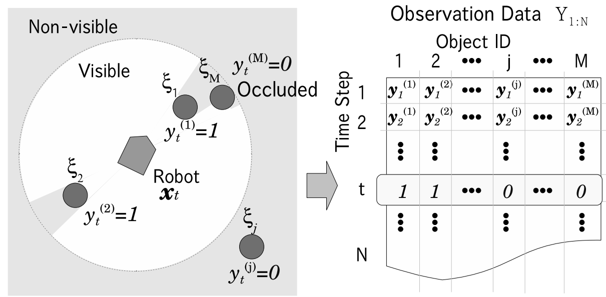

Figure 2. Visibility-Only Measurements

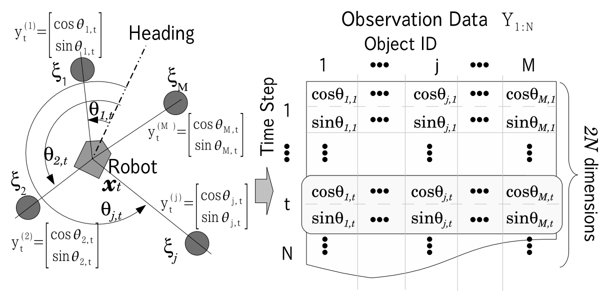

Figure 3. Bearing-Only Measurements

where $\delta_{i,j}$ and $w_{i,j}$ stand for the dissimilarity (Euclidean distance in our case) between i-th and j-th objects and corresponding weight, respectively. In this study, we consider two cases - (a) equal weights and (b) kNN-based weighting To avoid local minima, we used the solutions obtained by LPCA as initial solutions for SMACOF.

3. **Kernel PCA** (KPCA)(Scholkopf et al., 1998) : We tested (a) Gaussian (RBF) $k(\boldsymbol{a},\boldsymbol{b}) = exp(-\|\boldsymbol{a}-\boldsymbol{b}\|^2/(2\sigma^2))$ and (b) polynomial $k(\boldsymbol{a},\boldsymbol{b}) = (<\boldsymbol{a},\boldsymbol{b}> +1)^d$ kernels. σ^2 and d are parameters.
4. **ISOMAP**(Tenenbaum et al., 2000)
5. **Locally Linear Embedding** (LLE)(Roweis & Saul, 2000)
6. **Laplacian Eigenmap** (LEM)(Belkin & Niyogi, 2002)
7. **Hessian LLE** (HLLE)(Donoho & Grimes, 2003)
8. **Semi-definite Embedding** (SDE)(Weinberger et al., 2005)
ISOMAP, LLE, LEM, HLLE and SDE are graph-based manifold learning techniques. This time, we used k-nearest neighbors graphs, where k is the only parameter. In LEM, we assigned a weight $w_{i,j} = 1$ if i and j are connected on the graph, otherwise $w_{i,j} = 0$.

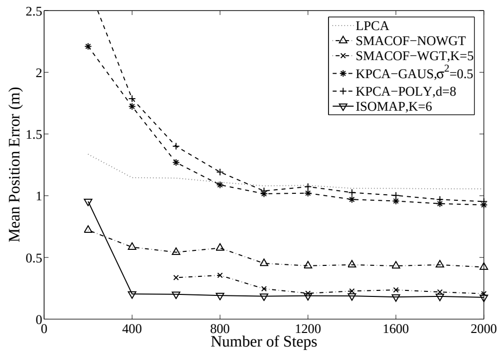

Figure 4. Transition of Mean Position Errors in Visibility-Only Mapping (LPCA,SMACOF,KPCA,ISOMAP)

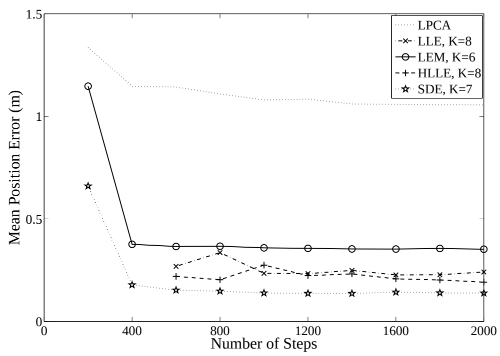

Figure 5. Transition of Mean Position Errors in Visibility-Only Mapping (LPCA,LLE,LEM,HLLE,SDE)

We used Matlab codes available publicly for ISOMAP, LLE, HLLE and SDE, and ones written by ourselves for the other methods (i.e., LPCA, SMACOF, KPCA and LEM).

4.3. Case 1 : Visibility-only Mapping

4.3.1. Problem Settings

A previous study on Visibility-Only Mapping (VOM) can be found in Yairi and Hori (2003). In VOM, a robot with a panoramic camera attempts to build a map using only *visibility* information, i.e., whether each objects is visible or not. In this simulation, it was assumed that the robot is able to recognize an object if its horizontal visual angle is larger than 5 degrees (0.0873 radian). In other words, each object is judged to be visible if it is within the distance of about 550 mm as long as it is not occluded.

In VOM, observation history data $\boldsymbol{Y}_{1:N}$ up to time N is represented as a M-by-N *binary* matrix each of

Table 1. Final Map Errors in Visibility-Only Mapping

DR methods	Opt. param.	MPE [m] (2000 stp)	MOE[%] (2000 stp)
LPCA(CMDS)	-	1.055	18.19
SMA(UNWGT)	-	0.421	5.86
SMA(WGT)	$K = 5$	0.206	4.83
KPCA(GAUS)	$\sigma^2 = 0.5$	0.926	23.29
KPCA(POLY)	$d = 8$	0.953	27.03
ISOMAP	$K = 6$	0.177	4.11
LLE	$K = 8$	0.241	5.40
LEM	$K = 6$	0.352	8.17
HLLE	$K = 8$	0.192	4.24
SDE	$K = 7$	0.138	3.65

whose elements is 0 or 1 (See also Figure 2). Therefore, applying LFMDR to the visibility-only mapping leads to a problem of embedding M N-dimensional binary vectors $\{\boldsymbol{y}_{1:N}^{(1)}, \cdots, \boldsymbol{y}_{1:N}^{(M)}\}$ into a 2-D plane. In this case, however, we used the *normalized* observation history vectors instead of the original ones as,

$$\tilde{\boldsymbol{y}}_{1:N}^{(j)} \equiv \boldsymbol{y}_{1:N}^{(j)} / \|\boldsymbol{y}_{1:N}^{(j)}\| \tag{7}$$

This normalization compensates the difference in how easily each object is recognized by the robot.

4.3.2. Results

Figure 4 and Figure 5 show how the map errors (MPEs) change according to the number of observations when each DR method is used. From these figures we can see the map accuracies are gradually improved as the number of observations increases. In other words, increasing the number of dimensions of observation history vectors leads to a better recovery of 2-D coordinates of the objects.

Table 1 summarizes the map errors (MPE and MOE) after 2000 observations (i.e., $N = 2000$) for the DR methods with their optimum parameter values. We notice that non-linear DR methods except KPCA outperform Linear PCA. This result is quite reasonable, because the assumption in the Visibility-Only Mapping that only nearby objects are visible is more favorable for the graph-based DR methods which focus on the local proximity of objects than LPCA which attempts to preserve the global covariance.

Figure 6 and Figure 7 show examples of maps obtained by applying LPCA and SDE to a set of observation data. In these figures, differences between true and estimated object positions are emphasized with lines. On the other hand, KPCAs with Gaussian and polynomial kernels are far behind the graph-based non-linear DR methods. This implies that not only nonlinearity

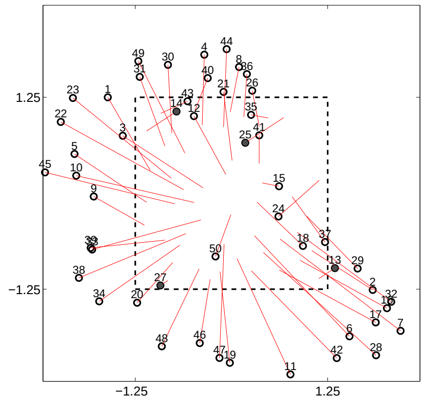

Figure 6. A Map Built by LPCA in Visibility-Only Mapping After 2000 steps (MPE:1.023m, MOE:15.95%)

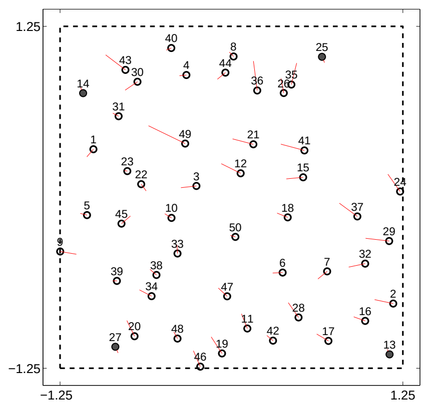

Figure 7. A Map Built by SDE in Visibility-only Mapping After 2000 steps (MPE:0.101m, MOE:2.77%)

but also locality should be taken into account in this problem.

Another noteworthy point is that SMACOF which is a relatively old technique performs as well as the latest spectral manifold learning methods, especially when the weights are assigned by k-nearest neighbors.

In terms of final map errors, SDE outperforms others, though it takes more computational time than others.

4.4. Case 2 : Bearing-only Mapping

4.4.1. Problem Settings

In recent years, there is an increasing interest in Bearing-Only SLAM (BOSLAM)(Deans & Hebert, 2000) that attempts to solve the SLAM problem using only a single inexpensive camera. In this section, we consider Bearing-Only Localization-Free Mapping (BOLFM) by applying the framework of LFMDR to the bearing measurements.

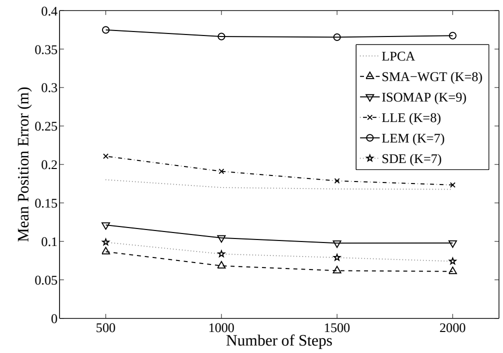

Figure 8. Transition of Mean Position Errors in Bearing-only Mapping (LPCA, SMACOF, ISOMAP, LLE, LEM, SDE)

In this experiment setting, the robot is able to measure the relative bearing angle to each object at each location on the path (See Figure 3). For example, let the relative bearing angle to j-the object at time t be $\theta_{t,j}$. We simply assume that the bearing measurements to all objects are always available, although it is unlikely in the real world due to various factors such as occlusion. If we had to take missing measurements into account, we could estimate them using EM algorithm. As in the previous subsection, we also assume that all objects are uniquely identifiable.

An issue when applying the LFMDR framework to the bearing measurements is the discontinuity between $-\pi$ and π. To avoid this problem, we used a unit directional vector $[\cos\theta_{t,j}, \sin\theta_{t,j}]^T$ as an observation $\boldsymbol{y}_t^{(j)}$ instead of the bearing angle $\theta_{t,j}$ itself. This makes the observation history vector seemingly $2 \times N$ dimensional.

4.4.2. Results

Figure 8 shows how MPE changes as the observation data increases in each DR method, and Table 2 lists MPEs and MOEs after 2000 observations for the DR methods. We omit the result of HLLE because it often failed to run in this experiment.

Compared with the results of VOM, maps are estimated with higher accuracy as a whole. This means that bearing measurements are more informative about spatial relationships between the robot and objects than visibility measurements.

A significant difference from VOLFM is that LPCA performs much better in BOLFM. In fact, surprisingly, the performance of LPCA was better than those of LLE and LEM in this case. We consider the reason for

Table 2. Final Map Errors in Bearing-Only Mapping

DR methods	Opt. param.	MPE [m] (2000 stp)	MOE[%] (2000 stp)
LPCA(CMDS)	-	0.168	2.33
SMA(UNWGT)	-	0.101	1.38
SMA(WGT)	$K = 8$	0.0609	1.00
KPCA(GAUS)	$\sigma^2 = 1.0$	3.47	49.2
KPCA(POLY)	$d = 2$	0.605	9.15
ISOMAP	$K = 9$	0.0979	1.83
LLE	$K = 8$	0.173	3.03
LEM	$K = 7$	0.367	8.46
SDE	$K = 7$	0.0741	1.36

this phenomenon is in the assumption that all objects are observable from every robot position, which decreases the locality of the observation history space. If we employed a different setting that distant objects are NOT observable, different results would be obtained.

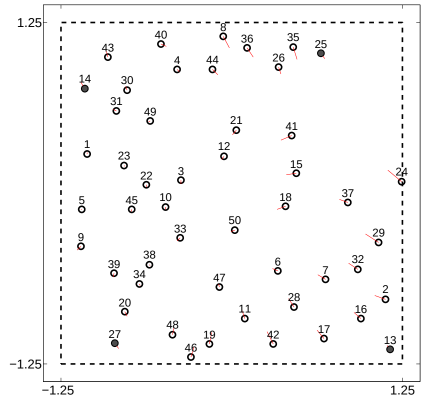

Figure 9. A Map Built by SMACOF (Weighted, K=8) in Bearing-Only Mapping After 2000 steps (MPE:0.0512m, MOE:1.10%)

5. Discussion

5.1. Robustness against Uncertainties

Although the proposed framework in the simplest form is vulnerable to the observation uncertainties such as measurement noise, indistinguishability of objects, and missing observations which are inevitable in the real world, a variety of enhancements to overcome the limitations can be considered. When some objects are not distinguishable accidentally, a simple data association can be performed by minimizing the loss function underlying a specific DR method with respect to not only the map of objects but also the correspondence between the measurements and the objects. When a relatively small portion of the observation data is missing due to occlusion and other non-structural noise, PCAMD (PCA with missing data) and PPCA using EM algorithm can be used. If the missing values occur in a structural way rather than randomly and their percentage is high, a promising approach will be to divide the whole set of objects and their observation data into overlapping subgroups, then build corresponding sub-maps and integrate them.

5.2. Scalability

Most of the DR techniques used in this work are not scalable to the data size (i.e., the number of objects in our case) in their original forms. However, several approximation methods such as Landmark-ISOMAP(de Silva & Tenenbaum, 2003) and Fast MVU(Weinberger et al., 2007) that make those DR techniques applicable to large-scale problems have been developed recently. Therefore, the easiest way to make the proposed framework scalable to the number of objects is to employ these approximation versions of DR methods. Another (but similar) approach is the divide-and-conquer approach (i.e., sub-map approach) mentioned above.

6. Conclusion

In this paper, we reconsidered the robotic map building from the viewpoint of dimensionality reduction (DR) of observation history data, and proposed a new framework called Localization-Free Mapping by Dimensionality Reduction (LFMDR). Unlike SLAM, LFMDR performs map building without localization, and does not require explicit models of state transition and observation. We also presented Visibility-Only Localization-Free Mapping and Bearing-Only Localization-Free Mapping , which are derived by applying this framework to visibility and bearing measurements, respectively. In the experiment, we tested them with several DR methods including linear and nonlinear ones. A variety of applications of LFMDR to other kinds of measurements such as Range-Only Mapping can be considered in a straightforward way.

There are, however, many issues to be solved such as how to deal with missing measurements, how to develop an online algorithm, and so on. Another interesting issue is how to integrate different information sources in this framework. For example, consider the situation that both visibility and bearing measurements are available. In that case, the problem of map building is to find a *common* low-dimensional representation for the two homeomorphic manifolds in their observation history spaces.

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